

# Nonlinear Optics and the Soliton Laser

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# Main Source of Information:

*Phil Trans. R. Soc. Lond. A* **315**, 437–460 (1985)

437

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## Solitons in optical fibres and the soliton laser

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In this paper, I describe both fundamental and higher-order solitons in optical fibres, their remarkable properties, and the first experimental observation of them. It will be shown that such solitons are easily created and, once formed, are quite stable in the one-dimensional world of single-mode fibres. Consequently, a number of exciting uses have already been found, or have been proposed for them.

One of those uses is in the soliton laser, a mode-locked (short-pulse) laser, whose pulse characteristics are determined by a length of single-mode fibre in its feedback loop. Pulse width scales with the square root of the fibre's length, in accord with  $N = 2$  soliton behaviour. The first version of this device, based on a colour-centre laser broadly tunable in the 1.5  $\mu\text{m}$  wavelength region, has already produced pulses as short as 0.13 ps. Compression in a second, external fibre has reduced those pulse widths to less than 50 fs, and reduction by at least another factor of two is considered likely in the near future.

# Outline

- Fiber-Optics

  - 1-D Fiber-Optics – A Soliton Playground

  - Solitons in Fiber-Optics – Why?

  - Solitons in Fiber-Optics – How?

- Nonlinear Schrödinger Connection

  - How we get to NLS from Fiber Optics

  - (Time) Periodic Solutions to NLS

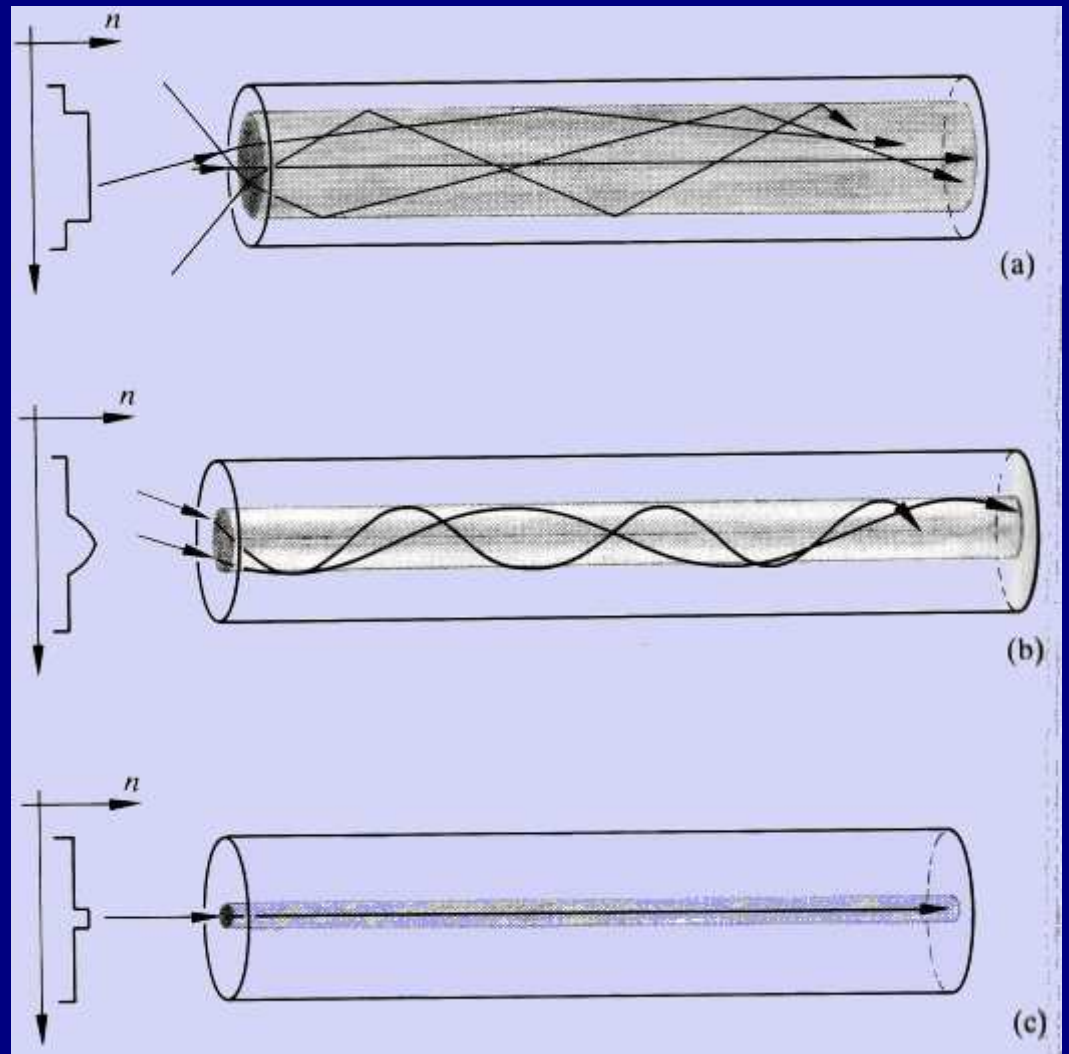
  - Experimental Observations

- The Soliton Laser

# 1-D Fiber-Optics – A Soliton Playground:

- (a) Multimode Step-Index Fiber
- (b) Multimode Graded-Index Fiber
- (c) Single-mode Step-Index Fiber

**SOLITONS!**



# 1-D Fiber-Optics – A Soliton Playground:

- No intermodal dispersion  
(narrow core  $\Rightarrow$  only one path is allowed)
- Crosssectional area  $\sim 10^{-6} \text{ cm}^2$   
 $\Rightarrow d \sim 11 \mu\text{m}$  (on the order of 10 wavelengths)
- Just like John Scott Russell's Canal...

# 1-D Fiber-Optics – A Soliton Playground:



# Solitons in Fiber-Optics – Why?

- Data transfer capabilities
  - copper telephone wires ~ 2 dozen conversations
  - mid-1980's pair of fibers ~12,000 conversations  
(equivalent to ~ 9 television channels)
  - early 1990's solitons in fibers ~ 70 TV channels  
(transmission rate of 4 Gb/s)
- Increase transmission rate, and distance between repeater stations

Statistics from: Hecht, *Optics*, (Addison-Wesley, New York, 1998)

# Solitons in Fiber-Optics - Why?

- Repeater Station distance determined by power loss:

$$\frac{P_o}{P_i} = 10^{-\alpha L/10}$$

- minimum  $\alpha = 0.16$  dB/km for fused silica fibers

$$P_o = 0.963829 P_i$$

- Repeater Station when power drops by  $\sim 10^{-5}$

$$\Rightarrow L \sim 300 \text{ km}$$

( $L \sim 1$  km for ordinary wire systems)



# Solitons in Fiber-Optics - Why?

- Soliton generation made possible by:  
Nonlinear index of refraction

$$n = n_0 + n_2 I$$

“Negative” group velocity dispersion

$$\frac{\partial v_g}{\partial \lambda} < 0$$

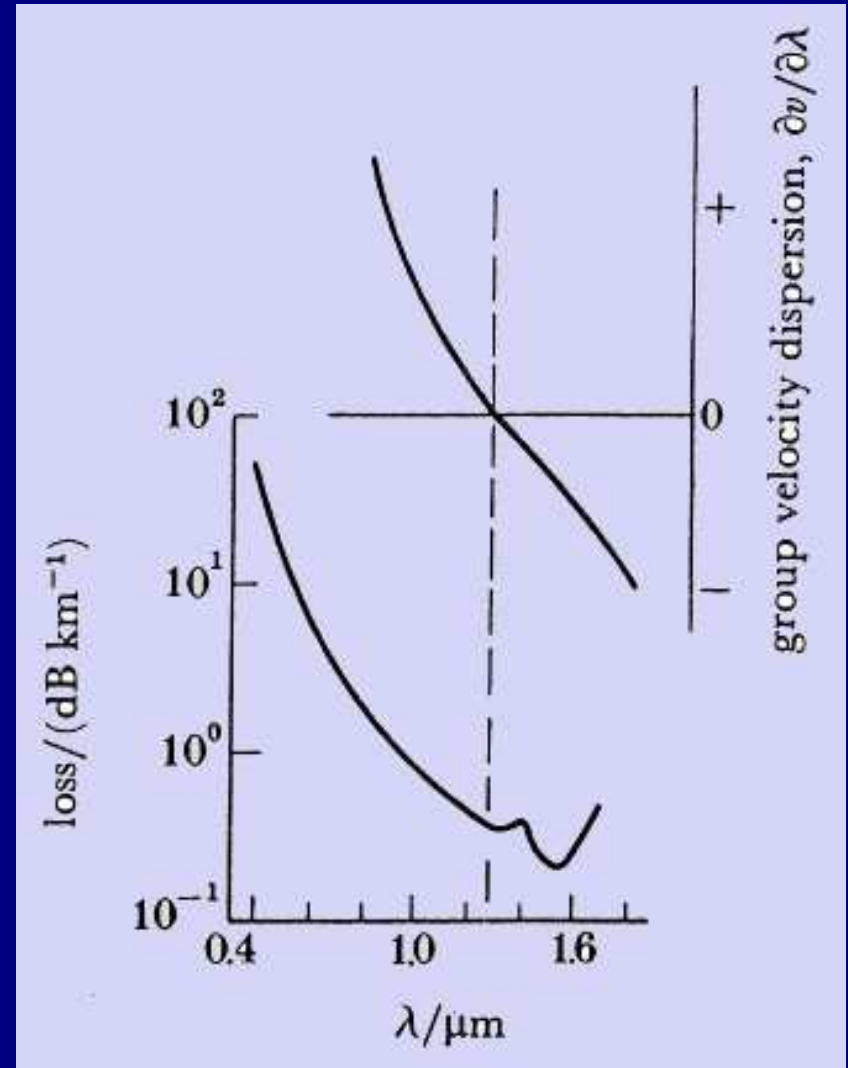
# Solitons in Fiber-Optics - Why?

- Note: region of negative dispersion includes region of minimum loss

- Operate with  $\lambda = 1.5 \mu\text{m}$   
(near IR in EM spectrum)

$$\alpha \approx 0.16 \text{ dB/km}$$

$$\frac{\partial v_g}{\partial \lambda} < 0$$



# Solitons in Fiber-Optics - How?

- Continuous wave will experience “self-phase modulation” due to nonlinear portion of index of refraction and length of fiber traversed:

$$\Delta\phi = \frac{2\pi}{\lambda} L n_2 I$$

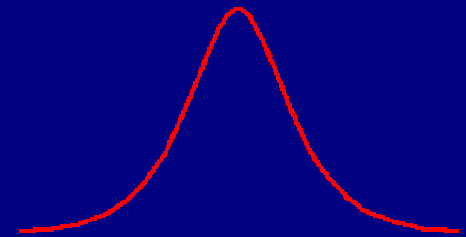
# Solitons in Fiber-Optics - How?

What happens to a pulse of light?

- **Self-Phase Modulation Effects:**

leading edge frequencies lowered

trailing edge frequencies raised

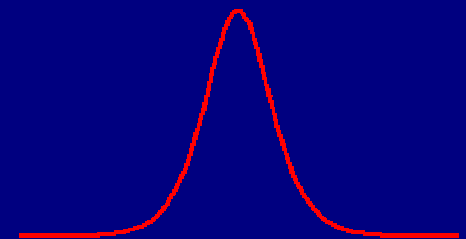


- **“Negative” Group Velocity Dispersion Effects:**

leading lowered frequencies slow down

trailing raised frequencies speed up

- $\Rightarrow$  **“Pulse Narrowing”**



# Solitons in Fiber-Optics – The Math

- In order to proceed from this qualitative concept of “Pulse Narrowing” we need Math
- $\Rightarrow$  The Nonlinear Schrödinger Connection

# Nonlinear Schrödinger Equation

- Assume light pulse can be expressed as:

$$M * u(z, t)$$

M: monochromatic term

u: envelope function of distance along fiber and time

- Envelope function satisfies:

$$i\left(\frac{\partial u}{\partial z} + k_1 \frac{\partial u}{\partial t}\right) = \frac{-k_2}{2} \frac{\partial^2 u}{\partial t^2} + \kappa |u|^2 u$$

where:  $k_1 = \frac{\partial k}{\partial \omega}$ ,  $k_2 = \frac{\partial^2 k}{\partial \omega^2}$ ,  $\kappa = \frac{k_0 n_2}{2 n_0}$

# Nonlinear Schrödinger Equation

- Perform the transformation

$$s = \frac{t - k_1 z}{\tau}, \quad \xi = \frac{|k_2| z}{\tau^2}, \quad v = \tau u \sqrt{\frac{\kappa}{|k_2|}}$$

- Obtain NLS in dimensionless form:

$$i \frac{\partial v}{\partial \xi} = \frac{1}{2} \frac{\partial^2 v}{\partial s^2} + |v|^2 v$$

# The N=1 Soliton Solution to NLS

- From initial data

$$v(0, s) = \text{sech}(s)$$



- Passes through the fiber unchanged
- Exact balance of dispersion and self-modulated pulse narrowing

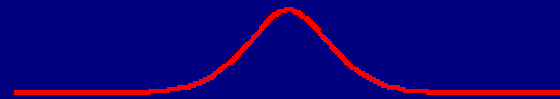


# The $N = 2$ Soliton Solution to NLS

- From initial data:

$$v(0, s) = 2 \operatorname{sech}(s)$$

- Periodic with period  $\pi/2$



- Envelope function:

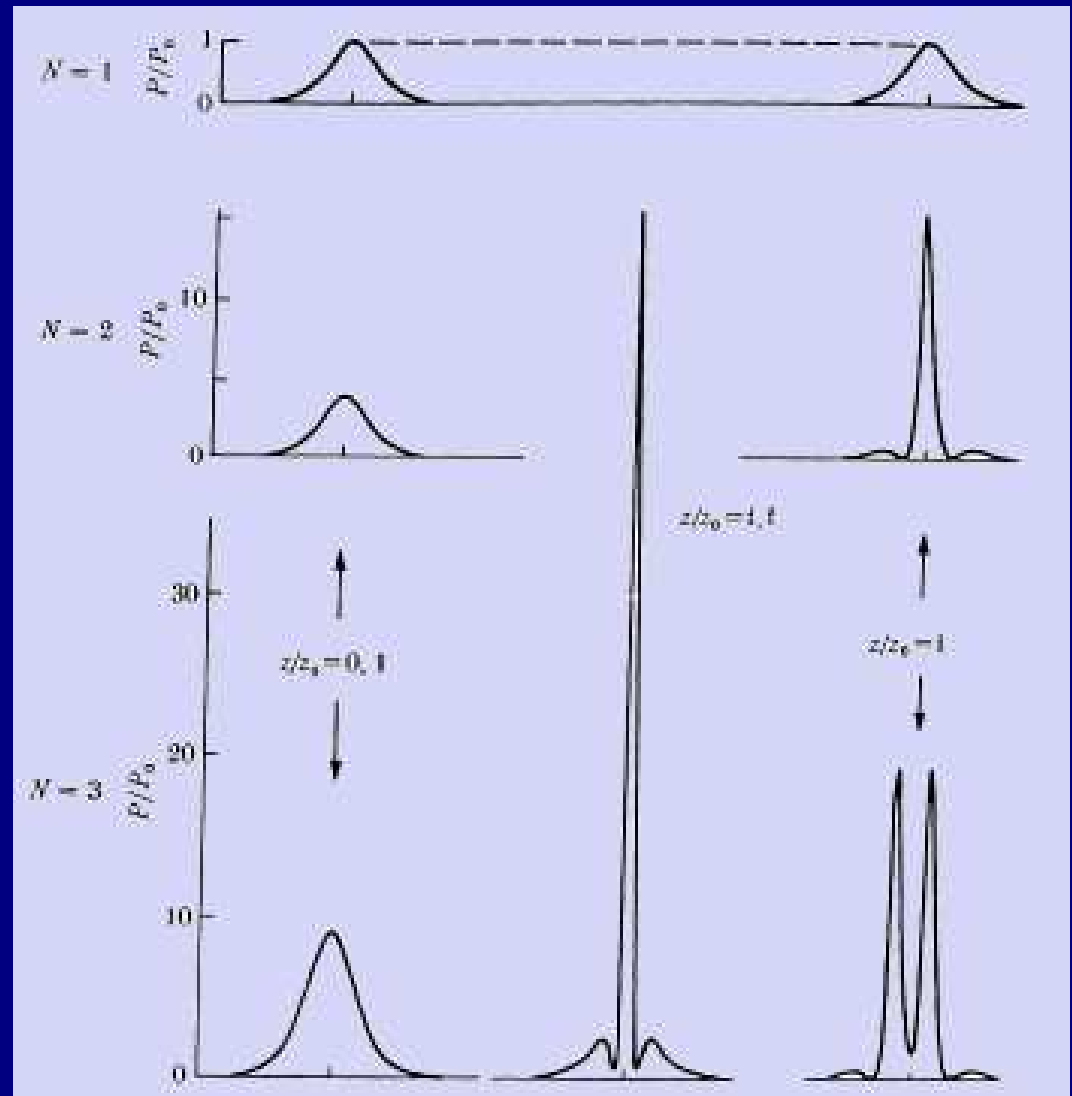
$$v(\xi, s) = \frac{4 e^{-i\xi/2} (\cosh(3s) + 3 e^{-4i\xi} \cosh(s))}{\cosh(4s) + 4 \cosh(2s) + 3 \cos(4\xi)}$$

- Intensity (plotted above – what we see)

$$I(\xi, s) = |v(\xi, s)|^2$$

# The $N > 2$ Soliton Solution to NLS

- From initial data  
 $v(0, s) = N \operatorname{sech}(s)$
- Periodic with  
period  $\pi/2$
- Complicated  
Envelope functions



# Fiber Experimental Setup

- Pass a  $\text{sech}^2$  shaped pulse through optical fiber:

$$L = \frac{\text{soliton period}}{2}$$

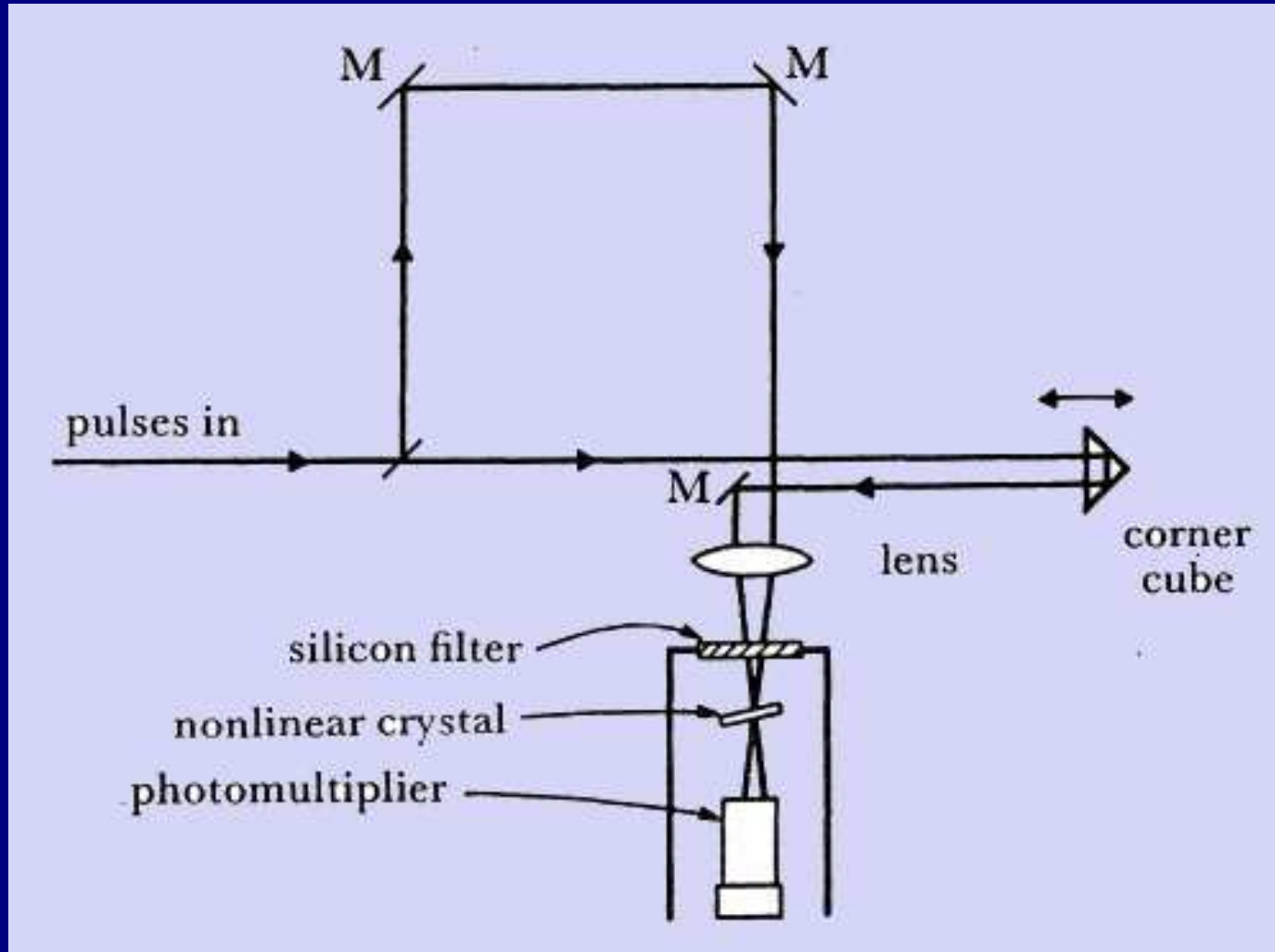
- Measure resulting pulse

(Using autocorrelation)

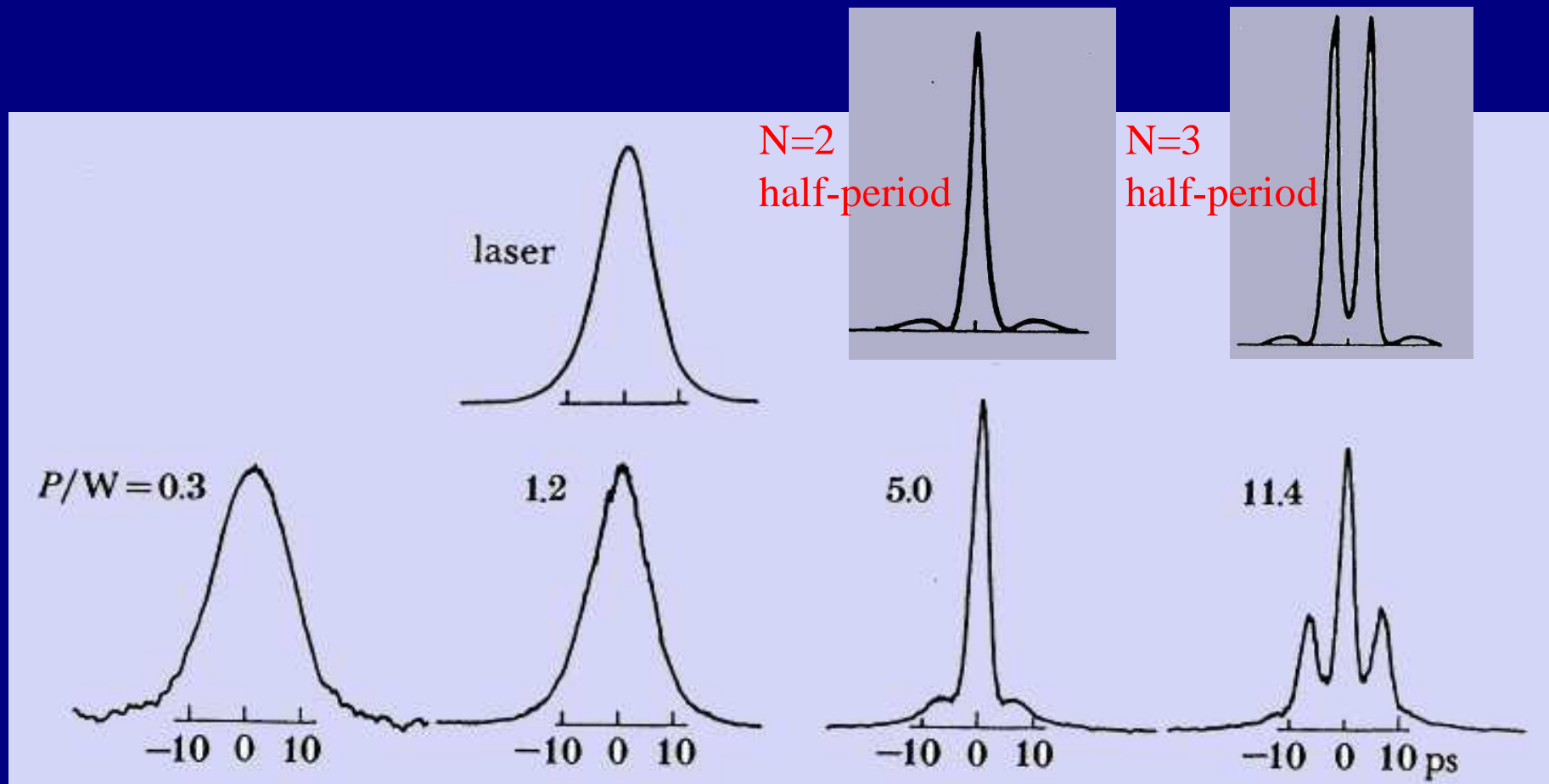
$$A(\tau) = \int_{-\infty}^{+\infty} I(t) \bar{I}(t - \tau) dt$$

# Fiber Experimental Setup

- Half-Period Fiber Output:



# Fiber Experimental Results



- Dispersion Dominated, N=1 balance, N=2 half-period behavior, N=3 behavior

# Autocorrelation Example:

- Intensity Profile  $I(t)$ :

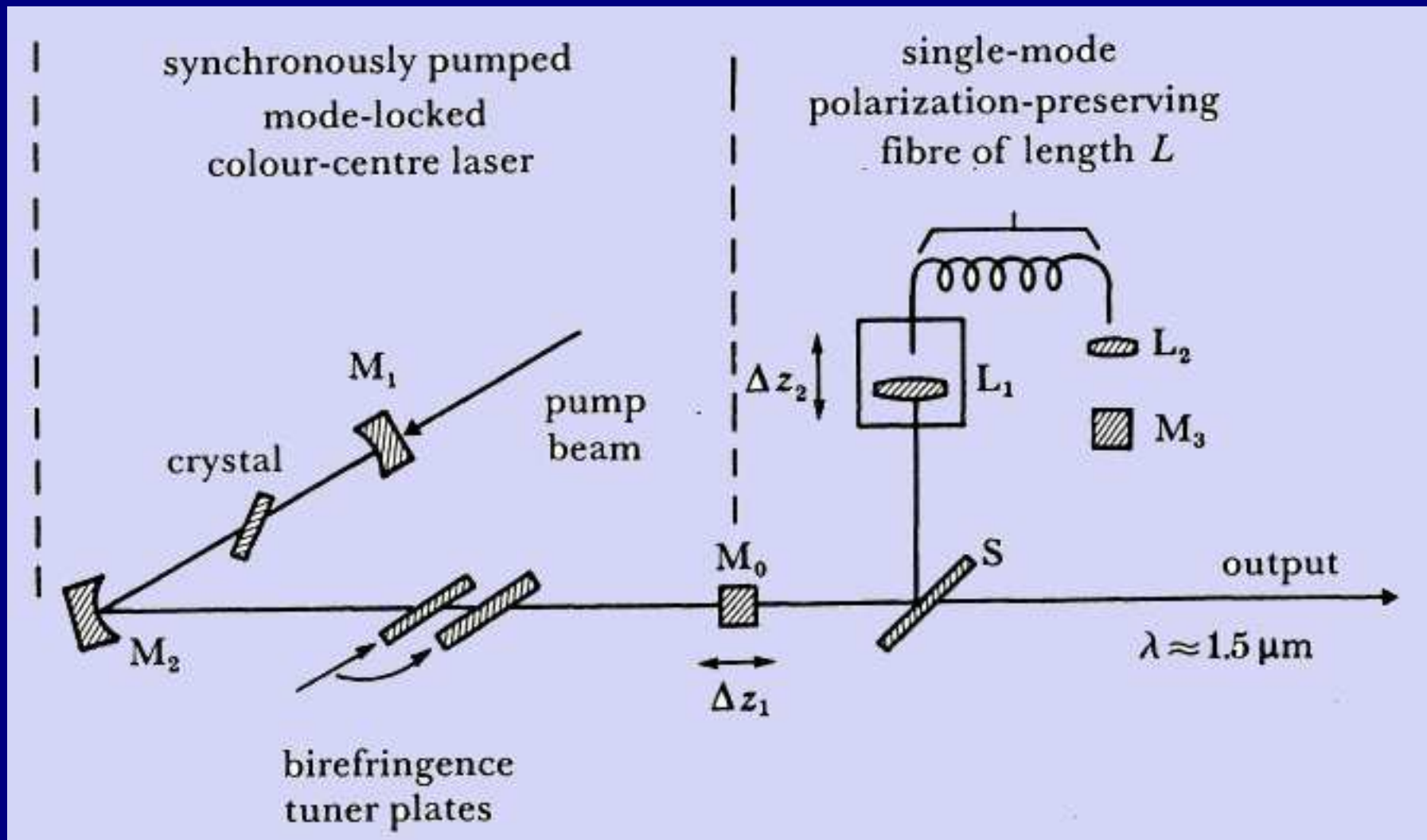
$$I(t) = \text{sech}^2(t-2) + \text{sech}^2(t+2)$$

- Autocorrelation Profile  $A(\tau)$ :

$$A(\tau) = \int_{-\infty}^{\infty} (\text{sech}^2(t-2) + \text{sech}^2(t+2)) (\text{sech}^2(t-2-\tau) + \text{sech}^2(t+2-\tau)) dt$$



# Soliton Laser - How?



# Soliton Laser Results

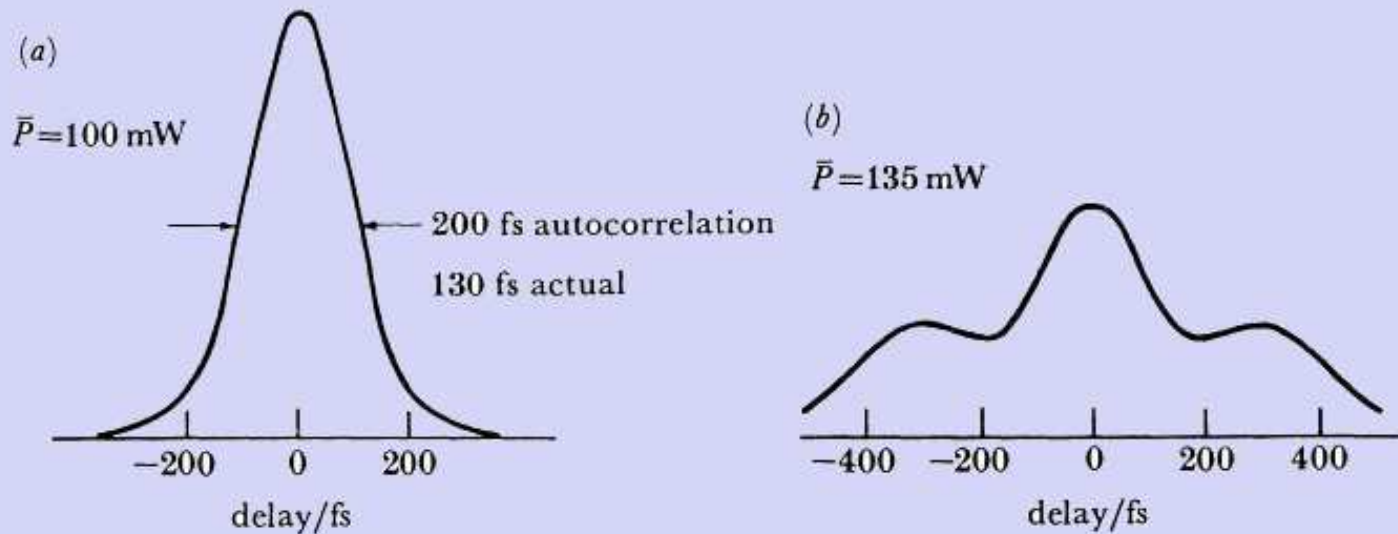


FIGURE 8. Autocorrelation shapes of: (a) typical 'best-shape' pulse; (b) pulse at higher fibre power. (See text.)

• Pulse In

• Pulse Out

⇒ N = 2 Soliton Behavior



# Sources:

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Questions?